

Table of Contents

1 Discovering Mathematics	1
1.1 What is Mathematics	1
1.2 Euclid's Elements	4
1.3 Finding the Circumference of a Circle	7
1.4 Golden Ratio and Fibonacci Sequence	8
1.5 Circular Functions	16
1.6 Exponents and Logarithms	19
1.7 Slopes and Derivatives	23
1.8 Areas and Integrals	32
1.9 Complex Numbers and Euler's Equation	39
1.10 Summary and More Discoveries	48
2 Discovering the World of Numbers	50
2.1 Counting and Number Systems	50
2.2 Arithmetic Series and Triangular Numbers	57
2.3 Squares and Square Roots	63
2.4 Multiplications and Geometric Series	66
2.5 Prime Numbers	70
2.6 Counting and Choosing	74
2.7 The Binomial Theorem	77
2.8 Recurrence Sequences	79
2.9 Modular Arithmetic	82
2.10 Continued Fractions	87
2.11 Public-key Cryptography	92
3 Discovering Trigonometry	99
3.1 Trigonometric Functions	99
3.2 Sine and Cosine Waves	100
3.3 Trigonometric Identities	102
3.4 Congruent and Similar Triangles	105
3.5 Area of Triangles	107
3.6 Power Series and Trigonometric Functions	110
3.7 Excel Plots of Trigonometric Functions	114
3.8 Inverse Trigonometric Functions	117
4 Discovering Probability and Statistics	121
4.1 Flipping a Coin	121
4.2 The Normal Distribution	129
4.3 The Central Limit Theorem	135
4.4 Rolling Dice	137
4.5 Surprising Probabilities	143
4.6 Conditional Probabilities	145
4.7 Bayesian Inference	147
4.8 Hypothesis Testing	148

5 Discovering Logic and Algorithms	153
5.1 Logical Operations	153
5.2 Logic Gates	155
5.3 Positive and Negative Logic: De Morgan's Theorem	158
5.4 Design of Digital Circuits	160
5.5 Boolean Algebra	164
5.6 Design of Multiplexers	169
5.7 Algorithms	173
5.8 A GCD Algorithm	175
5.9 A Sorting Algorithm	177
6 Discovering Algebra	183
6.1 The Quadratic Equation	184
6.2 Solving Linear Equations	191
6.3 Solving Cubic Equations	195
7 Discovering Geometry	201
7.1 Triangles and Circles	202
7.2 Conic Sections	205
7.3 The Volume of a Cone and a Sphere	213
7.4 Doubling the Cube	216
7.5 Trisecting an Angle	219
7.6 Fractal Geometry	220
8 Discovering Vectors	234
8.1 Addition and Subtraction of Vectors	236
8.2 The Scalar or Dot Product	243
8.3 The Vector or Cross Product	246
8.4 The Index Notation	251
8.5 Orthogonal Curvilinear Coordinate Systems	262
9 Discovering Calculus	270
9.1 The Fundamental Theorem of Calculus	271
9.2 Differentiation	277
9.3 Integration	287
9.4 Implicit Differentiation	299
10 Discovering Vector Differentiation	303
10.1 Linear Motion	303
10.2 Projectile Motion	306
10.3 Circular Motion	311
10.4 Plane Curvilinear Motion	316

11 Discovering Differential Equations	320
11.1 Another Look at the Exponential Function	320
11.2 Computer Solutions of Differential Equations	322
11.3 Analytic Solutions of Differential Equations	328
11.4 Second-Order Differential Equations	330
11.5 Linear Systems of Differential Equations	335
11.6 Eigenvalues and Eigenvectors	342
11.7 Classifying Linear Systems Solutions	344
11.8 A General Second-Order Differential Equation	359
11.9 Predator-Prey Differential Equations	362
11.10 Discovering Chaos	364
Index	375